

Mathematics Test Explanations

1. **The correct answer is B.** To find the midpoint of two points, you can take the average of the x and y coordinates. If point X has coordinates $(-4, 0)$ and point Y has coordinates $(0, -8)$, then the midpoint is:

$$\left(\frac{(-4 + 0)}{2}, \frac{(0 + -8)}{2} \right)$$

$$\left(\frac{-4}{2}, \frac{-8}{2} \right)$$

$$(-2, -4)$$

2. **The correct answer is K.** To solve, use the Pythagorean Theorem ($c^2 = a^2 + b^2$). Using values from $\triangle MNO$, you can set up the equation like this:

$$10^2 = 6^2 + NO^2$$

$$100 = 36 + NO^2$$

$$NO^2 = 100 - 36 = 64$$

$$NO = \sqrt{64} = 8$$

3. **The correct answer is C.** Since a distance in meters, M , can be approximated by multiplying a distance in yards, Y , by 1.0936, it follows that $M \approx Y(1.0936)$.
4. **The correct answer is H.** Because Seth has 4 plaid shirts and 5 solid-colored shirts, the total number of shirts is $4 + 5 = 9$. Of these 9 shirts, 4 are plaid. Thus the probability that a randomly selected shirt will be plaid is $\frac{4}{9}$.
5. **The correct answer is C.** Unless otherwise specified, average means “arithmetic mean,” which is defined as the sum of a set of values divided by the number of values. Therefore, you can see that the average number of enrollments per day is $\frac{(17 + 19 + 23 + 14 + 25 + 28)}{6} = \frac{126}{6}$, or 21.
6. **The correct answer is J.** When parallel lines are cut by a transversal (such as segment PR in this problem), “alternate interior angles” are congruent. In this problem, $\angle y$ and $\angle 4$ are alternate interior angles, and so you can conclude that they are congruent.
7. **The correct answer is B.** The easiest way to solve this problem is to take 25% of the original price and deduct it from the original price. To find 25%

of the sale price of the carton of paper, you would multiply \$27.00 by 25%, or 0.25. Therefore, the sale price of the carton of paper would be $\$27.00 - (\$27.00)(0.25) = \$27.00 - \6.75 , or \$20.25.

8. **The correct answer is H.** To solve this problem, you must recognize that parallel lines always have the same slope. Remember that to find the slope of the line, you have to convert the equation $2x - 3y = 7$ into slope-intercept form ($y = mx + b$, where m is the slope):

$$2x - 3y - 2x = 7 - 2x$$

$$\frac{(-3y)}{3} = \frac{(-2x + 7)}{(-3)}$$

$$y = \frac{-2x}{-3} + \frac{7}{-3}$$

$$y = \frac{2}{3}x - \frac{7}{3}$$

Thus the slope of this line, or any line parallel to it, is $\frac{2}{3}$.

9. **The correct answer is D.** In order to solve this problem, you must realize that if Andrew had \$28,000 remaining after paying 30% in taxes, then the \$28,000 constitutes $100\% - 30\%$ or 70% of the original prize, P . Therefore, $0.7P = 28,000$. Dividing by 0.7, you can conclude that the original cash value of the prize was $P = \frac{28,000}{0.7}$, or \$40,000.
10. **The correct answer is K.** To solve this problem, it is useful to assign values to the number of apples that Melissa and Marcia both possess. If Marcia has 10 apples, Melissa has $10 - 3 = 7$ apples. If Melissa gives 2 of her 7 apples to Marcia, Melissa is left with $7 - 2 = 5$ apples. When Marcia receives 2 more apples, she has $10 + 2 = 12$ apples. Since Marcia now has 12 apples and Melissa now has 5 apples, Melissa has $12 - 5 = 7$ fewer apples than Marcia.
11. **The correct answer is C.** The absolute value of a number is its distance from zero, regardless of whether it is positive or negative. Therefore, the value of $|5 - 9| = |-4| = 4$.
12. **The correct answer is K.** Since this problem requires you to multiply two binomials, you can utilize the FOIL (First, Outside, Inside, Last) method

to multiply the expressions.

$$\text{First: } (3m)(m^2) = 3m^3$$

$$\text{Outside: } (3m)(-n) = -3mn$$

$$\text{Inside: } (n)(m^2) = m^2n$$

$$\text{Last: } (n)(-n) = -n^2$$

Finally, add all these terms up to come up with your final answer. $(3m + n)(m^2 - n) = 3m^3 - 3mn + m^2n - n^2$.

- 13. The correct answer is A.** To solve this problem, you must distribute and add like terms, as follows:

$$13 - 2(x + 5) =$$

$$13 - 2x - 10 = 2x + 3$$

- 14. The correct answer is F.** Remember that the rule for exponents states that for base number b and exponents x and y , $(b^x)^y = b^{xy}$. Thus, when you apply the numbers from this problem, you find that $(n^7)^{11} = n^{(7)(11)} = n^{77}$.

- 15. The correct answer is B.** To solve this problem, recognize that the repeating decimal has four places (0.3456), and that the fourth place is occupied by the number 6. Therefore, every place that is a multiple of 4 will be represented by the number 6. Since 217 is not divisible by 4, you know that the 217th digit cannot be 6; eliminate answer choice E. Because 216 is a multiple of 4, the 216th digit will be 6. Therefore, the 217th digit must be 3, the next digit in the repeating decimal.

- 16. The correct answer is H.** If a square has side x , then its perimeter is $4x$; this is because a square is defined as a rectangle where all four sides are of equal length. Since the perimeter of the square is 48, then $48 = 4x$ and $x = \frac{48}{4} = 12$. Thus, the length of one side of the square is 12. The area of a square is defined as $(\text{side})^2$; therefore the area of this square is 12^2 or 144.

- 17. The correct answer is B.** The easiest way to solve this problem is to remember that when two binomial expressions are multiplied, there is a predictable result. Take the following generalized example: $(x + a)(x - b) = x^2 - bx + ax - ab$. If $x^2 - bx + ax - ab = 0$, then the solutions to the equation are $x = -a$ and $x = b$. The product of the solutions is $-ab$. With this expression, $x^2 + 3x - 21 = 0$, the product of the solutions $(-ab)$ is -21 .

- 18. The correct answer is H.** Remember that a difference of squares factors easily, such as: $a^2 - b^2 = (a + b)(a - b)$. Using the same technique, you can factor $a^{16} - 16$ into $(a^8 + 4)(a^8 - 4)$. The factor $(a^8 - 4)$ is another difference of squares, so it can be factored further into itself: $(a^8 - 4) = (a^4 + 2)(a^4 - 2)$. Of these factors, only $(a^4 + 2)$ is an answer choice.

- 19. The correct answer is E.** Recall that dividing by a fraction is equivalent to multiplying by the reciprocal. When $\frac{1}{4}$ is substituted for n in the following expression, $\frac{2n - 5}{n}$, the result is:

$$\begin{aligned} & \frac{2(\frac{1}{4}) - 5}{\frac{1}{4}} \\ &= \left(\frac{2}{4} - 5\right) 4 \\ &= 2 - 20 = -18 \end{aligned}$$

- 20. The correct answer is G.** Since 90 minutes is equal to 1.5 hours, a proofreader who can read 40 pages in one hour can read $(1.5)(40)$ or 60 pages in 1.5 hours.

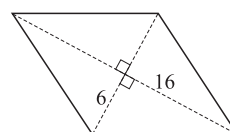
- 21. The correct answer is B.** The height, h , can be found using the Pythagorean Theorem ($c^2 = a^2 + b^2$):

$$\begin{aligned} 5^2 &= 3^2 + h^2 \\ 25 &= 9 + h^2 \\ h^2 &= 16, \text{ or } h = 4. \end{aligned}$$

Thus, when you multiply the base of the parallelogram by its height, the area of the parallelogram is $9 \times 4 = 36$.

- 22. The correct answer is F.** For a certain quadratic equation $ax^2 + bx + c = 0$, if $x = \frac{a}{b}$ is a solution, then a possible factor would be $(bx - a)$. Since two solutions for $ax^2 + bx + c = 0$ are $x = \frac{3}{4}$ and $x = \frac{-2}{5}$, then possible factors are $(4x - 3)$ and $(5x + 2)$.

- 23. The correct answer is B.** The diagonals of a rhombus intersect at their midpoints and form right angles as shown below.



Since the diagonals meet at their midpoints and form right angles, they form a right triangle with legs $\frac{12}{2} = 6$ and $\frac{32}{2} = 16$. To find the length of a side of the rhombus, you can simply use the Pythagorean Theorem and solve where the side of the rhombus, s , is the hypotenuse: $s^2 = 6^2 + 16^2 = 292$; s is approximately equal to 17.09.

- 24. The correct answer is J.** If a rectangular parking lot has a length, l , that is 3 feet longer than its width, w , then $l = 3 + w$, or $w = l - 3$. The area of a rectangle is equal to its length times its width, or $A = lw$. Since the area of this parking lot is 550, $lw = 550$. Substituting $(l - 3)$ for

$$550 = l(l - 3) =$$

$$550 = l^2 - 3l$$

$$l^2 - 3l - 550 = 0.$$

To solve for l , factor the quadratic equation to get $(l + 22)(l - 25) = 0$, making $l = -22$ or $l = 25$. Since negative values for length do not make sense in this context, the length is 25.

- 25. The correct answer is A.** To find the slope of the line between any two points (x_1, y_1) and (x_2, y_2) , you can use the equation $\frac{(y_2 - y_1)}{(x_2 - x_1)}$. Therefore, when you have the points $(3, 7)$ and $(4, -8)$ it follows that the slope of the line joining these points is $\frac{(-8 - 7)}{(4 - 3)} = \frac{-15}{1}$, or -15 .
- 26. The correct answer is G.** To find the solution set of $x + 2 > -4$, first solve for x by subtracting 2 from both sides. The result is $x > -6$. Thus the solution set is $\{x : x > -6\}$.
- 27. The correct answer is B.** To solve this problem, you need to know that the equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. Therefore, the center of the circle in the problem, $(x - 3)^2 + (y + 3)^2 = 4$, is $(3, -3)$.
- 28. The correct answer is G.** To find the distance between two points (x_1, y_1) and (x_2, y_2) , you can use the distance formula, which is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. The length of the line segment that has endpoints $(-3, 4)$ and $(5, -6)$ will equal the distance between points $(-3, 4)$ and

$(5, -6)$. Therefore, $d =$

$$\begin{aligned} & \sqrt{(5 - (-3))^2 + (-6 - 4)^2} \\ &= \sqrt{8^2 + 10^2} \\ &= \sqrt{64 + 100} \\ &= \sqrt{164} \\ &= \sqrt{(4)(41)} \\ &= \sqrt{4}\sqrt{41} \\ &= 2\sqrt{41} \end{aligned}$$

- 29. The correct answer is E.** The key to solving this problem is remembering that the triangle inequality states that no one side of a triangle can be greater than the sum of the other two sides. Thus the third side of the triangle in the problem cannot be greater than the sum of the other two sides, 4.7 and 9, which is 13.7. Of the answer choices, only 14 is too large to be a possible value for the third side of the triangle.
- 30. The correct answer is G.** To solve this problem, recall that $\frac{n^x}{n^y} = n^{x-y}$. Since it is given in the problem that $\frac{n^x}{n^y} = n^2$, you can conclude that $n^{x-y} = n^2$ and thus $x - y = 2$.
- 31. The correct answer is C.** To solve, convert the equation of the line to slope-intercept form ($y = mx + b$, where m is the slope and b is the y -intercept). If $3x + 5y = 8$, then:

$$3x + 5y - 3x = 8 - 3x$$

$$\frac{(5y)}{5} = \frac{(-3x + 8)}{5}$$

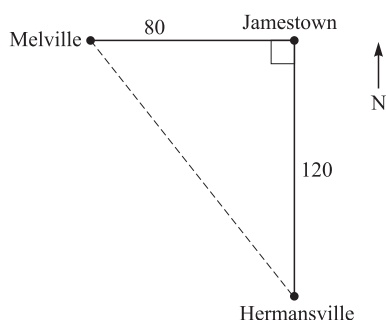
$$y = \frac{-3x}{5} + \frac{8}{5}$$

Since the equation $y = \frac{-3x}{5} + \frac{8}{5}$ is in slope-intercept form, the y -intercept is $\frac{8}{5}$.

- 32. The correct answer is F.** To find the cost per ounce, first convert 3.4 pounds to ounces. Because there are 16 ounces in a pound, 3.4 pounds is $3.4(16) = 54.4$ ounces. To find cost per ounce, divide the cost in dollars by the number of ounces, or $\frac{\$4.95}{54.4}$ ounces = \$0.09 per ounce.

- 33. The correct answer is C.** To solve, first square each fraction: $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$. Remember that to be added, fractions must have a common denominator. In this case, since 4 is a factor of 16, the lowest common denominator is $(9)(16) = 144$. To convert fractions into different denominators, you must multiply the top and bottom of a fraction by the *same* number. If $\frac{1}{4}$ is multiplied by $\frac{36}{36}$, the result is $\frac{36}{144}$. Likewise, multiplying $\frac{1}{9}$ by $\frac{16}{16}$ yields $\frac{16}{144}$, and multiplying $\frac{1}{16}$ by $\frac{9}{9}$ yields $\frac{9}{144}$. Therefore $\frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \frac{36}{144} + \frac{16}{144} + \frac{9}{144} = \frac{(36 + 16 + 9)}{144}$, or $\frac{61}{144}$.

- 34. The correct answer is G.** The easiest way to solve this problem is to draw a picture similar to the one below.



Since the route heads straight north from Hermansville for 120 miles to Jamestown, and then straight west for 80 miles to Melville, the turn at Jamestown creates a right angle. If a straight, flat road existed between Hermansville and Melville, it would form the hypotenuse of a right triangle with legs 80 and 120. Using the Pythagorean Theorem ($c^2 = a^2 + b^2$), you can see that the distance of this straight route from Hermansville to Melville would be:

$$\begin{aligned} & \sqrt{(120^2 + 80^2)} \\ &= \sqrt{(14,400 + 6,400)} \\ &= \sqrt{20,800} \approx 144 \end{aligned}$$

- 35. The correct answer is B.** To solve this problem, calculate the volume of the aquarium and divide by 2. Since volume is equivalent to length \times width \times height, the volume is $30 \times 16 \times 12$, or 5,760 cubic inches of water. Dividing by two, you see that half of the tank would be 2,880 cubic inches of water.
- 36. The correct answer is K.** To solve this problem, you would multiply the number of possibilities in each officer position. Since the league selects its 4 officers by first selecting the president, then the vice president, then the secretary, then the treasurer, there are 40 possibilities for president, 39 possibilities for vice president, 38 possibilities for secretary, and 37 possibilities for treasurer. The total number of different possibilities for the election is therefore $40 \times 39 \times 38 \times 37$.

- 37. The correct answer is A.** Because there is a right angle at S , the point T will lie along the line through S that is perpendicular to the segment RS . To solve this problem, find the equation for the line through S that is perpendicular to the segment RS and try each answer choice to find one that lies on the line. Since the line is perpendicular to segment RS , it will have a slope that is the opposite reciprocal of the slope of RS . Since slope is rise/run, the slope of RS is $\frac{(3 - 2)}{(6 - 2)} = \frac{1}{4}$. The slope of a line perpendicular to that is -4 . Because a point and the slope of the line are known, the point-slope form of the equation can be utilized. A line through point $(h, -k)$ with slope m has equation $y - k = m(x - h)$. Thus the line through $S(6, -3)$ that is perpendicular to the segment RS has equation $y - 3 = -4(x - 6)$. Distributing and adding like terms, the result is $y = -4x + 27$. Of the answer choices, only the point $(5, 7)$ falls on the line.

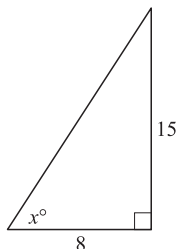
- 38. The correct answer is F.** To solve the equation $0.2(x - 2,700) = x$, first distribute:

$$0.2x - 540 = x$$

$$-540 = 0.8x$$

$$-675 = x$$

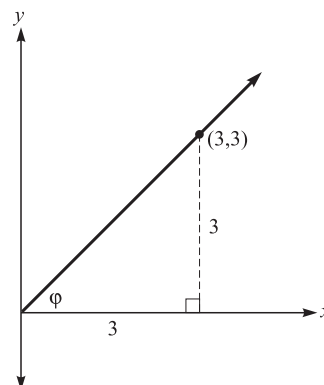
39. **The correct answer is A.** Given that $0^\circ \leq x \leq 90^\circ$ and that $\tan x = \frac{15}{8}$, x can be pictured in the right triangle below.



Because tangent is the ratio of the side opposite the angle to the side adjacent to the angle, the legs of the right triangle can be labeled as above. Cosine is the ratio of the side adjacent to the angle to the hypotenuse, which is not given. It is possible to eliminate answer choices in such a manner that it is not necessary to use the Pythagorean Theorem. Since the side adjacent to x is 8, the numerator in $\cos x$ will be 8, eliminating all but answers A and E. Since the legs of the triangle are 8 and 15, the hypotenuse will be longer than either, eliminating answer choice E. Thus $\cos x = \frac{8}{17}$.

40. **The correct answer is G.** Since the area of the square pool is given, you must find the area of the circle, with a radius of 10, and subtract the area of the pool. The area of a circle is equal to πr^2 , where r is the radius. The area of this circle is $10^2\pi = 100\pi \approx 314$ square feet. Thus the area of the enclosure is approximately $314 - 81 = 233$ square feet.
41. **The correct answer is B.** Remember that all parallel lines have the same slope, so a line parallel to $y = 2x + 2$ will have a slope of 2. A quick way to aid you in solving this problem would be to eliminate answer choices that do not have slope 2, so answer choices A and E can be immediately eliminated. Check the point $(3, -4)$ in the remaining answer choices. The only choice that works is $y = 2x - 2$.
42. **The correct answer is H.** Tangent is the ratio of the side opposite to the side adjacent to an angle in a right triangle. Drawing a line that passes through $(3, 3)$ and is perpendicular to the x -axis

creates a right triangle, as shown in the figure (see below).



Because point $(3, 3)$ is given, both legs of the right triangle have a length of 3. Thus $\tan \phi = \frac{3}{3} = 1$.

43. **The correct answer is A.** To solve, calculate the result for each operation and select the smallest result.

Answer Choice A: $\frac{2}{3} + (-3) = -2\frac{1}{3}$

Answer Choice B: $\frac{2}{3} - (-3) = 3\frac{2}{3}$

Answer Choice C: $\frac{2}{3} \times -3 = -2$

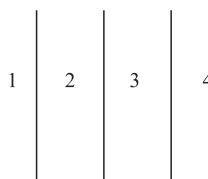
Answer Choice D: $\frac{2/3}{-3} = \frac{-2}{9}$

Answer Choice E: $\frac{[2/3 + (-3)]}{2} = \frac{-7}{6}$

The smallest result is $-2\frac{1}{3}$, which was obtained by adding.

44. **The correct answer is F.** To simplify calculations, you can multiply the entire equation by 12 to obtain whole numbers and get $4b + 24 = 3$. Subtracting 24 from both sides yields $4b = -21$. Dividing by 4 yields $b = -\frac{21}{4}$, which is a little less than -5 . Thus the correct answer will lie between -4 and -6 .
45. **The correct answer is B.** To find the solution set for $|3a - 2| \leq 7$, break it up into two separate inequalities: $3a - 2 \leq 7$ and $3a - 2 \geq -7$. Starting with $3a - 2 \leq 7$, solving for a yields $a \leq 3$. With $3a - 2 \geq -7$, solving for a yields $a \geq -\frac{5}{3}$. Thus a is between $-\frac{5}{3}$ and 3 inclusive.

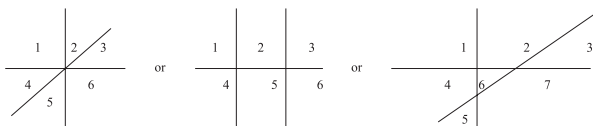
- 46. The correct answer is K.** Tangent is the ratio of the side opposite to the side adjacent to an angle in a right triangle. If the distance, in feet, to the cell phone tower is x , then $\tan 41^\circ = \frac{200}{x}$, or $x = \frac{200}{\tan 41^\circ}$. Since $\cot 41^\circ = \frac{1}{\tan 41^\circ}$, $x = \frac{200}{\tan 41^\circ} = 200 \cot 41^\circ$.
- 47. The correct answer is A.** Since the area of a square is equal to the square of its sides, multiplying the sides by $\sqrt{3}$ will have the effect of multiplying the area by $(\sqrt{3})^2 = 3$.
- 48. The correct answer is H.** In order to solve this problem, you must realize that since the volume of a cube is equal to the cube of its sides, multiplying the length of the sides by $\frac{1}{2}$ will have the effect of multiplying the volume by $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$. The cube in this problem has a volume of 64, so if you halve the length of each side, new cube's volume will be $64\left(\frac{1}{8}\right) = 8$.
- 49. The correct answer is D.** The area of a parallelogram is equal to base \times height. In the figure, you can see that the base of the parallelogram is 7 and the height of the parallelogram is 9. Thus, the area of the parallelogram is $9 \times 7 = 63$.
- 50. The correct answer is G.** In order for $8a^6b^3$ to be less than zero, either 8 or a^6 or b^3 must be less than zero. However, it is obvious that $8 > 0$ and any number taken to an even power is non-negative. Thus $b^3 < 0$ and in order for that to be true, $b < 0$. Of the answer choices, only $b > 0$ CANNOT be true.
- 51. The correct answer is D.** Logarithms are used to indicate exponents of certain numbers called bases. By definition, $\log_a b = c$, if $a^c = b$. If $\log_4 x = 3$, then $x = 4^3$, or 64.
- 52. The correct answer is J.** In order for a system of 2 linear equations to have no solutions, the graphs of the equations must be parallel. Parallel lines have the same slope. To find the equation whose graph is parallel to the line in the figure, you must find the slope of the line between the points $(0, -4)$ and $(3, -0)$. Since slope is $\frac{\text{rise}}{\text{run}}$, the slope is $\frac{4}{3}$. The only equation with the correct slope of $\frac{4}{3}$ is $y = \frac{4}{3}x + 2$.
- 53. The correct answer is C.** Of the 80 marbles, only 8 end in 5. If the first marble is drawn and not replaced, there are 79 marbles left, 7 of which have a ones digit of 5. Thus the probability that the player will be a winner is $\frac{7}{79}$.
- 54. The correct answer is H.** To solve this problem, remember that the formula for slope is equal $\frac{(y_2 - y_1)}{(x_2 - x_1)}$, where (x_1, y_1) and (x_2, y_2) are two given points on a line. The equation of the line that passes through the origin and the point $(3, 4)$ will have slope $\frac{(4 - 0)}{(3 - 0)} = \frac{4}{3}$. Since the line passes through the origin, the y -intercept is 0. Thus the correct equation is $y = \frac{4}{3}x$.
- 55. The correct answer is C.** To solve this problem, you must remember that in an isosceles triangle, the base angles have the same measure. Since the sum of angles is 180° for all triangles, $180 = (a + 30) + 2(2a - 15)$. Distributing and adding like terms yields
- $$180 = (a + 30) + 4a - 30$$
- $$180 = 5a$$
- $$a = 36$$
- Since the base angles are equivalent to $2a - 15$, they equal $2(36) - 15 = 72 - 15$, or 57° .
- 56. The correct answer is K.** The smallest possible value will occur when it is negative. A negative product will result only when one of the numbers is positive and one is negative. The possible pairs are then -1 and 6 , -2 and 5 , -3 and 4 , -4 and 3 , -5 and 2 , and -6 and 1 . Of these pairs, the smallest product is $(-3)(4) = (-4)(3)$, or -12 .
- 57. The correct answer is A.** Start by drawing 3 parallel lines.



This creates 4 distinct regions, so the minimum number of distinct regions must be 4. Eliminate answer choices C, D, and E.

Now, try drawing 3 lines in other configurations, and you will see that there will always be either

6 or 7 regions:



Therefore, the correct answer is 4, 6, or 7 distinct regions, answer choice A.

- 58. The correct answer is F.** Remember that the area of a parallelogram is equal to base \times height. In this case, the base is $[3 - (-3)]$ or $[2 - (-4)]$, both of which equal 6, and the height is $(3 - (-5)) = 8$. Thus the area is $6 \times 8 = 48$. The area of triangle QRS , $\left[\frac{1}{2}(b)(h)\right]$, is half the area of the parallelogram, or 24.

- 59. The correct answer is E.** Refer to the following chart to follow the patten of the sequence.

Term	1	2	3	4	...	n
Term	a	ab	ab^2	ab^4	...	ab^{n-1}

Since the power of b is one less than the number of term, the n th term will be ab^{n-1} . The 643rd term will then be $ab^{643-1} = ab^{642}$.

- 60. The correct answer is H.** Since AB is longer than BC , there are only two possible configurations: B is between A and C or C is between A and B . In the case that B is between A and C , $AC = AB + BC = 19 + 13 = 32$. In the case that C is between A and B , $AC = AB - BC = 19 - 13 = 6$. Therefore, AC can be 6 and 32 only.